RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2017

FIRST YEAR [BATCH 2016-19]

Date : 22/05/2017 **MATHEMATICS FOR INDUSTRIAL CHEMISTRY (General)**

Time : 11 am – 2 pm **Paper :** II Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Unit-I

Answer **any three** questions from **Question Nos. 1 to 5** : [3 X 5]

1. a) Show that the points P(1, 5, –1), Q(0, 4, 5), R(–1, 5, 1) and S(2, 4, 3) are coplanar. 2
   b) Prove by the vector method that the straight line joining the middle points of two sides of a triangle is parallel to the third side and is also half of its length. 3

2. a) Let $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}$, $\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$. Find $\vec{a} \times (\vec{b} \times \vec{c})$ in terms of $\hat{i}, \hat{j}, \hat{k}$ and then verify $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. 3
   b) If the volume of a tetrahedron be 2 units and three of its vertices are $A(1, 1, 0)$, $B(1, 0, 1)$, $C(2, -1, 1)$ then find the locus of the fourth vertex. 2

3. The lines L_1 and L_2 have their vector equations given by $\vec{r} = 3\hat{i} + \hat{j} + t(2\hat{j} + \hat{k})$,
   $\vec{r} = 4\hat{k} + s(\hat{i} + \hat{j} - \hat{k})$
   where s, t are scalars. Show that L_1 and L_2 intersects and find the equation of the plane containing them. 5

4. Find the point of intersection of the three planes $\vec{r} \cdot \vec{n}_1 = p_1$, $\vec{r} \cdot \vec{n}_2 = p_2$, $\vec{r} \cdot \vec{n}_3 = p_3$ where $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors, not necessarily unit vectors. 5

5. A force of 15 units acts in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ and passes through a point $2\hat{i} - 2\hat{j} + 2\hat{k}$. Find the moment of the force about the point $\hat{i} + \hat{j} + \hat{k}$. 5

Unit-II

Answer **any five** questions from **Question Nos. 6 to 14** : [5 X 5]

6. Prove that a sequence of real numbers can not converge to more than one limit. 5

7. a) Show that the sequence $\{a_n\}$, where $a_n = \dfrac{(3n+1)(n-2)}{n(n+3)}$ converges to a finite limit. 3
   b) Show that the series $\sum_{n=1}^{\infty} \dfrac{n}{2^n}$ converges. 2

8. Examine the convergence or divergence of the series $x + \dfrac{1}{2} \cdot \dfrac{x^3}{3} + \dfrac{1 \cdot 3}{2 \cdot 4} \cdot \dfrac{x^5}{5} + \cdots (x > 0)$. 5

9. a) If a function $f(x)$ satisfies the conditions of the Mean Value Theorem and $f'(x)=0$ for all $x\in(a,b)$, then prove that $f(x)$ is constant on $[a,b]$. 3

b) Evaluate: $\lim_{x\to 0}\dfrac{xe^x-\log(1+x)}{x^2}$ 2

10. If a function *f* is such that its derivative f' is continuous on $[a,b]$ and derivable on (a, b), then show that there exists a number *c* between *a* and *b* such that $f(b)=f(a)+(b-a)f'(a)+\dfrac{1}{2}(b-a)^2 f''(c)$. 5

11. Expand the function $f(x)=\sin x$ by Maclaurin's Theorem. 5

12. Evaluate: $\lim_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$. 5

13. Examine the function $f(x)=\sin x+\cos x$, for extreme values. If exists, then find it. 5

14. Using the Lagrange's method of undetermined multiplier, show that the maximum value of the function $f(x,y)=xy$ subject to the condition $5x+y=13$ is $\dfrac{169}{20}$ and it is attained at $x=\dfrac{13}{10},\ y=\dfrac{13}{2}$. 5

Unit-III

Answer **any two** questions from **Question Nos. 15 to 17** : [2 X 5]

15. a) Evaluate: $\int_0^{\pi/2}\dfrac{dx}{3+5\sin x}$. 4

b) Prove that $\int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx$, if $f(a+x)=f(x)$. 1

16. Evaluate: $\lim_{n\to\infty}\sum_{r=1}^{n}\dfrac{n^2}{\left(n^2+r^2\right)^{3/2}}$. 5

17. Obtain the reduction formula for $\int\tan^n x\,dx$ and hence find the value of $\int\tan^6 x\,dx$. 5

Group – B

Answer **any five** questions from **Question Nos. 18 to 24** : [5 X 5]

18. State the axiomatic definition of probability. Deduce the classical rule from the axiomatic definition. 2+3

19. Under what conditions is the binomial distribution approximated as Poisson? Derive the variance of Poisson distribution. 2+3

20. State and prove Bayes' theorem in probability. 2+3

21. The download time of a resource web page is normally distributed with a mean of 6.5 seconds and a standard deviation of 2.3 seconds. What is the probability that the download time will be between 4 and 10 seconds? 5

22. The probability that a driver must stop at any one traffic light is 0.2. There are 15 sets of traffic lights on the journey. (a) What is the probability that he must stop at exactly 2 of the 15 sets of traffic lights? (b) What is the probability that he will be stopped at 1 or more of the 15 sets of traffic lights? 2+3

23. Find the probability that a leap year chosen at random will contain 53 Mondays. 5

24. Let the random variable X have the following pdf

$$f(x) = cx(2-x), \quad 0 \le x \le 2$$
$$= 0 \quad , \text{ otherwise}$$

Find c and calculate the probability $P(0.5 \le X \le 1)$. 3+2

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